CE Systems – Linear Programming – 2014

1. An Excel Solver sensitivity report for a linear programming model is given below. **INTERPRET ALL** of the information given for **decision variable "C"** (Adjustable Cells Table) and **constraint "C&D"** (Constraints Table). Use your words! ☺ The more complete your answer, the better prepared you will be for a similar question on an exam.

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B1	A	0	4	8	1E+30	4
B2	B	10	0	3	4	1E+30
B3	C	0	3	7	4	1
B4	D	5	0	6	1	4

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B6	A&B	10	4	10	5	10
B7	C&D	20	7	20	1E+30	15
B8	B&D	15	-1	15	15	5

Solution:

Decision Variable C:

- Final Value is 0, i.e., the optimal solution incorporates no C.
- Reduced Cost is 3, i.e., forcing C from 0 to 1 increases the optimal value of Z by 3 units.
- Objective coefficient is 7, A is multiplied by 7 in the objective function
- Allowable Increase is 4, the objective coefficient for C can be increased by 4 without changing the location (the binding constraints) of the optimal solution. Any additional increase will change the location.
- Allowable decrease is 1, the objective coefficient for C can be decreased only 1 unit without changing the location of the optimal solution. Any additional decrease will change the location.

Constraints C&D:

- Final Value is 20, this is the L.H. Side of the constraint resulting from the final values of the decision variables present on the L.H. Side.
- Shadow Price is 7, increasing the R.H. Side of the constraint one unit will increase Z by 7 units in the optimal solution.
- Constraint R.H. Side is 20. Because this equals the Final Value, this constraint is binding.
- Allowable Increase is infinity, the R.H. Side can be increased by infinity without changing the location of the optimal solution.
- Allowable decrease is 15, the R.H. Side can be decreased by 15 without changing the location of the optimal solution. Any additional decrease will change the location.

2. Use Solver to find the optimal solution to the Linear Programming problem given below. Report the values of Z, A, B and C in the optimal solution. State and **interpret** any non-zero Reduced Costs and Shadow Prices. Which constraints are binding?

Min Z = 3A + 10B + 4C
Subject to:
$A + B \geq 40$
$2B + C \geq 35$
All decision variables are non-negative.

Solution:

- Max Z = 242.5, A = 22.5, B = 17.5, C = 0.
- Forcing C to be one will increase Z by 0.5 units in the optimal solution. Increasing the R.H. Side of the first constraint will increase Z by 3 units. Increasing the R.H. Side of the second constraint will increase Z by 3.5 units.
- Both constraints are binding.

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B15	A	22.5	0	3	7	1
B16	B	17.5	0	10	1	7
B17	C	0	0.5	4	1E+30	0.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
B19	A + B	40	3	40	1E+30	22.5
B20	2B +C	35	3.5	35	45	35

CE Systems – Linear Programming – 2014

3. A company installs and removes erosion control fence at construction sites (to keep sediment from washing off-site during rainfall). Minimize the cost of installing fence for the month described below. Use Excel Solver. When old fencing is removed it is inspected, repaired, and cleaned before it can be considered refurbished; thus, it is not available until the next week. Furthermore, it must be used by the second week of storage. This means that fence removed during week 1 can only be reused in weeks 2 and 3, etc. At the start of the month you have 3000 feet of refurbished fence stored in its second week it can be used. Company practice dictates that you must have at least 3000 feet of refurbished fence available at the end of the month, i.e., that could be used in the first week of the next month. Installing fence costs $2/feet (new), $1/feet (refurbished used in first possible week), or $1.25/ft (refurbished used in second possible week). I used 10 decision variables, N_1, N_2, N_3, N_4, $R_{1.2}$, $R_{2.1}$, $R_{3.1}$, $R_{3.2}$, $R_{4.1}$, $R_{4.2,}$ 8 constraints (4 of which were equalities) not including non-negativity, and obtained a minimum cost of $38,875. N = New; R = Refurbished; the subscript is the week; and ".1" and ".2" are added to the subscript to indicate refurbished fence in the first or second week it can be used, respectively.

Week	Fence to be removed (feet)	Fence to be installed (feet)
1	6000 (of which 6000 can be refurbished)	8000
2	8000 (of which 5500 can be refurbished	7000
3	7000 (of which 3500 can be refurbished	5000
4	6000 (of which 2000 can be refurbished	7500

Use the format I used in class to present your model (see problem B above). Include a printout of your Excel Spreadsheet, showing the optimal solution. Arrange it like the Brick example (linked from assignments page). Also include the sensitivity report. **Explain** the two non-zero reduced costs, the 1.75 shadow price, and the -0.75 shadow price for the constraint with $R_{1.2}$.

Solution (8 constraints + non-negativity):
Objective Function: Min $Z = 2(N_1 + N_2 + N_3 + N_4) + 1.25(R_{1.2} + R_{3.2} + R_{4.2}) + 1(R_{2.1} + R_{3.1} + R_{4.1})$

Decision Variables: N_1, N_2, N_3, N_4, $R_{1.2}$, $R_{2.1}$, $R_{3.1}$, $R_{3.2}$, $R_{4.1}$, $R_{4.2}$ (feet of new or reused fence installed each week)

Subject to:

$N_1+R_{1.2}$	=	8000
$N_2+R_{2.1}$	=	7000
$N_3+R_{3.1}+R_{3.2}$	=	5000
$N_4+R_{4.1}+R_{4.2}$	=	7500
$R_{1.2}$	≤	3000
$R_{3.2}+R_{2.1}$	≤	6000
$R_{4.2}+ R_{3.1}$	≤	5500
$R_{4.1}$	≤	2500

All decision variables are non-negative.

Interpretations (See sensitivity report, attached):

- Forcing N_3 to 1 changes $R_{3.1}$ to 4999, $R_{4.2}$ to 1 & N_4 to 4499 at net cost of 0.25.
- Forcing $R_{3.2}$ to 1 forces $R_{2.1}$ to 5999, N_2 to 1, R3.1 to 4999, $R_{4.2}$ to 501, and N_4 to 4499 at a net cost of 0.5.
- Increasing the RHS of the third constraint one unit increases Z by 1.75 dollars ($R_{3.1}$ goes up 1, $R_{4.2}$ down 1, and N_4 up 1).
- Increasing the RHS of the fifth constraint one unit decreases Z by 0.75 dollars (can substitute one foot of refurbished fence in its second week of storage for a foot of new fence).

CE Systems – Linear Programming – 2014

Problem C continued

Spreadsheet Printout

Objective Function						
z	38875					
Decision Variables			Subject to:			
n1	5000		n1+r1.2	8000	=	8000
r1.2	3000		n2+r2.1	7000	=	7000
n2	1000		n3+r3.1+r3.2	5000	=	5000
r2.1	6000		n4+r4.1+r4.2	7500	=	7500
n3	0		r1.2	3000	≤	3000
r3.1	5000		r3.2+r2.1	6000	≤	6000
r3.2	0		r4.2+r3.1	5500	≤	5500
n4	4500		r4.1	2500	≤	2500
r4.1	2500					
r4.2	500					

Problem C continued

Sensitivity Report Printout **2 point**

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B28	n1	5000	0	2	1E+30	0.75
B29	r1.2	3000	0	1.25	0.75	1E+30
B30	n2	1000	0	2	1E+30	0.5
B31	r2.1	6000	0	1	0.5	1E+30
B32	n3	0	0.25	2	1E+30	0.25
B33	r3.1	5000	0	1	0.25	1E+30
B34	r3.2	0	0.5	1.25	1E+30	0.5
B35	n4	4500	0	2	0.25	0.75
B36	r4.1	2500	0	1	1	1E+30
B37	r4.2	500	0	1.25	0.75	0.25

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
E28	n1+r1.2 B20	8000	2	8000	1E+30	5000
E29	n2+r2.1 B20	7000	2	7000	1E+30	1000
E30	n3+r3.1+r3.2 B20	5000	1.75	5000	500	4500
E31	n4+r4.1+r4.2 B20	7500	2	7500	1E+30	4500
E32	r1.2 B20	3000	-0.75	3000	5000	3000
E33	r3.2+r2.1 B20	6000	-1	6000	1000	6000
E34	r4.2+r3.1 B20	5500	-0.75	5500	4500	500
E35	r4.1 B20	2500	-1	2500	4500	2500